AN INTRODUCTION TO RANDOM DYNAMICAL SYSTEMS AND THEIR PERTURBATIONS.

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Prerequisites to the course. Calculus in one and more variables, understanding of the basic concepts of analisys in $\mathbb{R}^n$. Basic Linear Algebra (good undesrstanding of the basic concepts). Basic topology in metric spaces (mainly about the concepts of: continuity, open, closed, compact sets).

Overview. The course aims to show an approach to the study of the statistical properties of deterministic and random dynamical systems trough the transfer operator method. We will see that using this approach and some basic functional analysis we can get deep results on interesting classes of systems.

We apply the methods to dynamical systems with noise, wich constitute a very flexible class of systems, in which we can find realistic models many important natural and social phenomena. We will present theoretical results and reliable numerical methods for the study of the statistical properties of these systems with a main focus on the stability of the statistical properties on perturbations of the system and on its linear response (the direction of change of these properties) this topic is relatively new in the mathematical scenario but has many potential applications. A class of questions to which the methods we introduce can answer is:

- What is the the expected time average of a given observable in some given system?
- What fluctuations we may expect?
- What if the system changes slowly during time? can I compute the increasing or decreasing rate of the average of a given observable?
- How can one (optimally) control the statistical properties of the system?

The course is aimed to introduce the main mathematical ideas about the transfer operator method in order to answer the questions outlined above, triying to reduce and simplify the formalism as much as possible.

The theoretical part of the course will be followed every afternoon by an "examples and exercises" session, where we will use software implemented for the purpose of studying the statistical properties of dynamical systems. We recommend each student to participate with his personal computer. The software we use is based on Sagemath and Cocalc computing platforms (https://cocalc.com/) whose use does not require deep programmation skills. The basic information needed to use these platforms however will be given in the course.

Date: March 4, 2019.

1. Content of the course (theoretical part)

Introduction to the subject. (1h)

Examples of random and deterministic dynamical systems and of what we are interested to understand about their statistical behavior. (1 h)

Recalls from measure theory and functional analysis.(2h)

We are going to recall the main ideas on: Metric and normed spaces. Measure theory, Lebesgue integral, L^1, L^2 spaces, Bounded Variation functions. Linear operators between normed vector spaces and their basic properties.

Basic Ergodic theory for deterministic systems (2h)

Systems preserving a measure, ergodic and mixing systems, the pointwise ergodic theorem, examples of chaotic and non chaotic systems.

The transfer operator method for deterministic systems.(4h)

The transfer operator. The right functional spaces to consider. Regularization estimates. Existence of regular invariant measures. Physical measures and their properties. Convergence to equilibrium and the spectrum of the transfer operator. Central limit, asymptotic laws. What happen when we perturb the system? Examples.

Random systems (2h)

Random systems, their transfer operators. Classes of regularizing random systems with noise. Regularization estimates and the right functional spaces to consider. Convergence to equilibrium and the spectrum of the transfer operator. Central limit, asymptotic laws.

Perturbations and linear response (2h)

Linear response of regularizing Markov operators, application to systems with additive noise. Examples.

Control of the statistical properties.

2. Content of the course (examples and exercises part)

Introduction to Sage (2h)

Finite elements methods for the study of dyamical systems (2h)

Introduction to the Ulam method, examples. Self validated numerical methods.

Examples of rigorous computation of stationary measures in deterministic and random systems (2h)

Examples of rigorous computation of other statistical properties (2h)

References

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[3] Lasota, A.; Mackey, M. C., *Probabilistic Properties of Deterministic Systems.* Cambridge University Press 1986.

[4] Lasota, A.; Mackey, M. C. *Chaos, Fractals, and Noise: Stochastic Aspects of Dynamics* Applied Mathematical Sciences Springer (1998)

[5] Liverani C *Invariant measures and their properties. A functional analytic point of view* Pubbl. Mat. Centro de Giorgi Scuola Normale Superiore, Pisa (2003)

[6] Viana M. *Lectures on Lyapunov Exponents*, Cambridge Studies in Advanced Mathematics 145, Cambridge University Press (2014)

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