Summer School of Mathematics for Economic and Social Sciences, San Miniato

Mathematical Methods for Time Series Analysis
Marco Lippi

1. **Time series.** Stationary stochastic processes. Autocovariance function. White noise processes. Moving averages. Infinite moving averages. Prediction. Wold Representation Theorem. ARMA processes.

2. **Vector time series.** Extension of the standard definitions to the n-dimensional case: stationarity, autocovariance function, etc. Wold Representation Theorem. VARMA processes. Topics 1 and 2 are covered in Hamilton (1994). (Although I go over stochastic processes, stationarity, ARMA processes etc., I assume some previous knowledge of these topics.)

3. The spectral representation of stationary stochastic processes. Very important but little known approach to time series. Requires complex numbers and functions. A complete but not easy reference is Brockwell and Davis (1987). Notes will be available.

4. Dynamic Factor Models in Macroeconomics.

4.1 Early dynamic factor models: Sargent and Sims (1977), Geweke (1977). Based on the idea that macroeconomic variables are driven by a small number of common factors, Burns and Mitchell (1946), these papers consider models with a large-but-not-too-large number of variables. They employ frequency domain methods.

4.2 High-dimensional factor models, static: Chamberlain and Rothschild (1983), Chamberlain (1983). Here the model has an infinite number of variables. They introduce the asymptotic definition of idiosyncratic components, allowing for weak correlation across the variables.

4.3 High-dimensional factor models, dynamic: (a) Time-domain approach, Stock and Watson (2002a), Stock and Watson (2002b). Here the estimators are based on the standard principal components. (b) Frequency domain, Forni et al. (2000), Forni and Lippi (2001). The estimators are based on the principal components of the spectral density.

4.4 Determining the number of factors: Bai and Ng (2002), Hallin and Liška (2007).

4.5 Forecasting: Stock and Watson (2002a), Stock and Watson (2002b), Boivin and Ng (2005), D'Agostino and Giannone (2012), Forni et al. (2018).

4.6 Identification of the factors: Stock and Watson (2005), Forni et al. (2009). These papers establish a correspondence between factor models and SVAR analysis.

References

Bai, J. and Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191–221.

Boivin, J. and Ng, S. (2005). Understanding and comparing factor-based forecasts. *International Journal of Central Banking*, 1(3).

Brockwell, P. J. and Davis, R. A. (1987). *Time series: theory and methods*. Springer-Verlag: New York.

Burns, A. F. and Mitchell, W. C. (1946). *Measuring Business Cycles*. NBER.

Chamberlain, G. (1983). Funds, Factors and Diversification in Arbitrage Pricing Models. *Econometrica*, 51(5):1281–1304.

Chamberlain, G. and Rothschild, M. (1983). Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets. *Econometrica*, 51(5):1305–1324.

D'Agostino, A. and Giannone, D. (2012). Comparing alternative predictors based on large-panel factor models. *Oxford Bulletin of Economics and Statistics*, 74(2):306–326.

Forni, M., Giannone, D., Lippi, M., and Reichlin, L. (2009). Opening the black box: structural factor models with large cross sections. *Econometric Theory*, 25(05):1319–1347.

Forni, M., Giovannelli, A., Lippi, M., and Soccorsi, S. (2018). Dynamic factor model with infinite dimensional factor space: Forecasting. *Journal of Applied Econometrics,* forthcoming.

Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2000). The generalized dynamic-factor model: Identification and estimation. *Review of Economics and Statistics*, 82(4):540–554.

Forni, M. and Lippi, M. (2001). The generalized dynamic factor model: Representation theory. *Econometric Theory*, 17(6):1113–1141.

Geweke, J. (1977). The dynamic factor analysis of economic time series. In Aigner, D. J. and Goldberger, A. S., editors, *Latent variables in socio-economic models*, volume 103, pages 365–383. North-Holland.

Hallin, M. and Liška, R. (2007). Determining the number of factors in the general dynamic factor model. *Journal of the American Statistical Association*, 102(478):603–617.

Hamilton, James, D. (1994). *Time series Analysis*. Princeton University Press, Princeton.

Sargent, T. J. and Sims, C. A. (1977). Business cycle modeling without pretending to have too much a priori economic theory. In Sims, C. A., editor, *New methods in business cycle research*. Federal Reserve Bank of Minneapolis.

Stock, J. H. and Watson, M. W. (2002a). Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association*, 97(460):1167–1179.

Stock, J. H. and Watson, M. W. (2002b). Macroeconomic forecasting using diffusion indexes. *Journal of Business & Economic Statistics*, 20(2):147–162.

Stock, J. H. and Watson, M. W. (2005). Implications of dynamic factor models for VAR analysis. NBER Working Paper 11467, NBER.