Learning outcomes: On completion, a student will be able to

• Demonstrate programming proficiency and skills in turning mathematical equations and models into working code.

• Solve practical problems in financial mathematics, code simple agent-based models in economics and estimate model parameters applying modern numerical techniques.

Content:

• Introduction to programming languages, programming basics, data types, operators, expressions, control structures, vector/array operations, input/output, plots, programming style, floating-point representation of real numbers, numerical errors, stability and conditioning of numerical algorithms, grids, histograms, efficient use of computational resources, examples from linear algebra and from quadrature.

• Random numbers: simple and more advanced generators for the standard uniform distribution, requirements and statistical tests, pathologic cases; inversion, transformation and acceptance-rejection methods to obtain other distributions in one and more dimensions, correlated normal random variates, quasi-random numbers.

• Monte Carlo 1: link between parabolic partial differential equations and stochastic differential equations, Euler-Maruyama algorithm, approximation error, strong and weak solution, Milstein algorithm, simulation of Lévy and stochastic volatility processes, pricing of European options with Monte Carlo.

• Monte Carlo 2: simple agent-based models of the distribution of wealth in a closed conservative economy with different random exchange rules and homogeneous or heterogeneous agents, asymptotic results and comparison with empirical data.

• Fourier transform: definitions, inverse transform, properties, notable transform pairs, discrete and fast Fourier transform, characteristic function, correlation/convolution theorem, auto/cross-covariance and correlation, Plancherel and shift theorems, pricing of European options in Fourier space.

• Optimization problems and model calibration/parameter estimation in normal and Fourier space: Newton-Raphson method, more advanced local and global gradient-based optimisation methods in several dimensions, constraints, implied volatility, equivalent forms, attraction basin, fractals.

• Exotic options: Fourier transform methods for the numerical pricing of discretely and continuously monitored path-dependent options (barrier,hindsight, etc.).