September 11-15, 2023.
Short Course on:

Dynamic Optimization in Economics and Finance

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Timing of Lectures

Textbook: Notes from the teachers on their websites

Tentative time table (6 hours per day: total 30 hours):

From Monday, 11 September 2023 till Friday, 15 September 2023:

– Morning: 9.30-11 and 11.30-13.00;

– Afternoon: 14.30-16.00 and 16.30-18

Morning Lectures will be mainly devoted to theory and examples.

Afternoon Lectures will be mainly devoted to questions and exercises in an interactive way: for example the teacher will suggest some exercises to do leaving some time to do them and then discuss them at the blackboard.

Outline of the course

Day 1:

Introduction to the Dynamic Optimization in discrete and continuous time; Dynamic Programming in discrete time.

- Examples of Dynamic Optimization problems in Economics:
    - Basic ones: utility maximization, optimal investment, optimal portfolio;
    - Recent ones: climate change and economics, control of infectious diseases, pollution.
- Dynamic optimization problems as Optimal Control Problems: mathematical setting in discrete and continuous time: state equation, pointwise constraints, set of admissible control strategies, objective functional, optimal strategies, optimal state paths, value function.
- Discounted autonomous infinite horizon problems.
- Feedback control strategies: admissible and optimal feedback maps.
- Dynamic Programming (DP) Principle and Bellman equation in discrete time, finite and infinite horizon.
- Optimality conditions via DP: verification theorem.
- Guess-and-verify method and examples.
- Solution of Bellman equation through fixed point theorems.

Day 2:

Dynamic Programming in continuous time. HJB equations and viscosity solutions.

- Dynamic Programming in continuous time: value function, dynamic programming principle, HJB equation: the finite horizon case and the infinite horizon autonomous case with discount. Examples.
- Optimality conditions via DP: verification theorem.
- Guess-and-verify method and examples.
- Some basics on Viscosity Solutions Theory for HJB equations and Examples.

Day 3:

Maximum Principle in discrete and continuous time.

- Pontryagin Maximum Principle (PMP) in discrete time: the finite horizon case.
- PMP in the infinite horizon case: transversality conditions. Examples.
- PMP in continuous time, finite horizon.
- PMP in continuous time, infinite horizon. Transversality conditions. Saddle path stability.
- Examples of applications.

Day 4:

Existence and uniqueness of optimal strategies

Some basic tools of functional analysis: Banach and Hilbert spaces, weak topologies and convexity.

Existence theorems trough compactness.

Uniqueness theorems through strict convexity/concavity or through verification theorems

Day 5:

Introduction to stochastic control in discrete and continuous time

Motivating examples.

Some ideas on controlled Stochastic Difference/Differential equations

The discrete time case: Bellman equations and verification theorem

The continuous time case: Ito formula, HJB equations and verification theorem.

Application to optimal portfolio problems

Main exercises of the course

- Check if a given control strategy is admissible
- Check if a given feedback map is admissible
- Given a Dynamic Optimization problem translate it as an Optimal Control Problem in standard form.
- Prove that the value function is finite.
- Given a Dynamic Optimization problem in discrete/continuous time write the necessary conditions of the maximum principle and, in simple cases, try to solve them.
- Solve simple Dynamic Optimization problems in discrete/continuous time and in finite/infinite horizon with the Dynamic Programming method, writing the Bellman/HJB equation and solving it.