Probability and Statistical Mechanics in Information Science
June 3 - July 20, 2003

Information Theory, as developed by Shannon, is a fundamental theory behind all of Digital Communications. The theory gives a precise statement about the fundamental limitations to reliable communication over noisy channels. One of the crowning achievements of this theory is the **Noisy Channel Coding theorem**, which defines a quantity called capacity of the channel based on maximization of mutual information between the input and the output of the channel over all input probability distributions. If transmission takes place at a rate below capacity then reliable transmission in a precise sense can take place. Conversely, reliable transmission cannot take place if messages are transmitted at a rate above capacity. The proof of the theorem requires taking a suitable **thermodynamic** limit (block sizes of the codes going to infinity). **Capacity achieving codes** have now been designed based on low density parity check codes. The research period at the Centro E. De Giorgi focused on recent progresses and future perspectives on the relations between these subjects and other branches of probability theory and statistical mechanics, notably disordered systems, belief propagation, large deviations and concentration inequalities. New heuristic and rigorous results on coding can be obtained using these techniques.

The second topic addressed in the period was that of **networks: information theory, queuing, relations with percolation, random matrices, spanning trees and random graphs**. Today, we live in a networked world and communication takes place in a networked environment with the Internet providing a universal communication environment to which various other networks (for instance wireless networks) are attached. An Information and Communication Theory in the Shannon's sense, suitable for a networked environment, is currently not available but there are promising researches in progress. In recent works on the subject there are suggestions that continuum percolation may be relevant. There is also intense activity on Multi User Information Theory where nested lattice codes may have a role to play. However, in order to emulate Shannon's idea, we have to take a suitable thermodynamic limit and there will not be a universal way of doing this. Moreover, recent work in Probability Theory such as random walks in random environment, stochastic geometry of spanning trees, random matrices and the related free probability theory are likely to play a role in Network Information Theory. The aim of the research period at the Centro De Giorgi was to promote interaction between experts of stochastic networks, queuing theory, information theory, percolation and other subjects of probability and statistical mechanics, with the hope to contribute to the development of a Network Information Theory.

Many other topics of Information Science relate to probability and statistical mechanics. We intended to touch at least a few questions on quantum computing and other computational ideas related to physics and networks.

The period was characterized by several short series of basic/advanced lectures on information theory, probability, networks, disordered systems, usually organized in intensive weeks. The first main aim was to give the opportunity to Ph.D. and Post Doc students to learn advanced subjects and address their researches. The second main purpose was to allow researchers from different areas to meet and discuss the frontiers of these subjects.

Scientific Committee

S.K. Mitter (M.I.T.)
G. Parisi (Roma "La Sapienza")
S.R.S. Varadhan (Courant Institute)

Organizing Committee

S. Caracciolo (Milano)
F. Fagnani (Torino)
F. Flandoli (Pisa)
A. Mennucci (Scuola Normale Superiore)
The period was organized in weeks devoted to special topics:

1. Large deviation, random matrices and disordered systems, communication, 02 June - 07 June
2. Quantum information theory (chair: Caracciolo, Fagnola, Pavon), information and control, 03 9 June - 14 June
3. Statistical mechanics, coding, and computation (chair: Tatikonda), 16 June, 2003 - 21 June
4. Non-equilibrium statistical mechanics and spin glasses, 23 June - 28 June
5. Network information theory and statistical mechanics (especially percolation theory), 30 June -5 July
6. Scaling limits (chair: Dai Pra, Isopi, Morato), 07 July - 12 July
7. Network information theory, 14 July - 19 July.

The first week was mainly devoted to rigorous probabilistic results applied to statistical mechanics and information theory.

Gérard Ben Arous gave three lectures (one for the E. De Giorgi Colloquium) on rigorous results for the dynamics of disordered systems. The audience has been introduced the concept of aging, still very new to the mathematical public. Aging is a property of dynamics in complex disordered media. A system ages when its decorrelation properties are age-dependent: the older the system the longer it takes to forget its past. The few systems for which aging has already been proved have been reviewed in the lectures. They are either mean-field like the REM or spherical SK model, or some trap model like the random walk with random rates, where aging takes place only in dimension one and, in a weaker form, in dimension two.

Ofer Zeitouni gave three lectures on the theory of random matrices applied to a problem of communication through a noisy environment. He started from a problem of signal estimation in a multi input-output communication framework, where N receivers get signals linearly dependent form K inputs, coupled together, with additive gaussian noise. Each bit is sent over a random signature to allow for a reconstruction. The optimal linear estimate can be formulated by means of a random matrix and its behavior is described by certain properties of the spectrum of such matrix. Under suitable assumptions on the randomness of the matrix, the empirical distribution of the spectrum has a scaling limit; Zeitouni described first its concentration properties based on concentration inequalities, then he treated the difficult topic of the large deviations for the spectrum, where he employed a dynamical approach based on matrix valued diffusion-like processes. He obtained a form of the rate functional and an Euler equation associated to it.

In the same week **Olivier Leveque** gave a talk on network information theory providing asymptotic results about the concepts of capacity and rate of transmission in wireless networks. **Martin Hairer** gave also a talk on ergodic properties of systems driven by long memory processes, based on suitable formulations of ergodic theory in such a non-Markovian context and using coupling arguments.

The second week was addressed to quantum information theory and was organized by **Sergio Caracciolo**, **Franco Fagnola** and **Michele Pavon**.

It opened with the engineering oriented talk of **Carlo Cosmelli** who first overviewed the most important open questions for the construction of a quantum computer and then described the existing advances and first technologies. One of the emerging mathematical problems with practical relevance is the control of quantum systems, the possibility to construct quantum gates and prepare a system in a given quantum state.

Several talks have been then devoted to this subject. First, **Michele Pavon**, after an introduction to stochastic calculus and in particular to Nelson stochastic mechanics, analyzed in detail the problem of quantum Schroedinger bridge and described solutions of it based on stochastic representations of the quantum states.

Ugo Boscain considered the optimal control of a low dimensional quantum system with the purpose of getting explicit results and described a practical application.

Claudio Altafini, on the other side, gave an overview of controllability results for quantum systems and related Lie algebraic conditions, summarized essentially in the fact that controllability is a general property for finite dimensional quantum systems, but there are basic obstacles to have it for open systems described by quantum Markovian master equations.

Other aspects of quantum information theory have been addressed by the other speakers. **Luigi Accardi** gave three lectures on different very important topics: the first one on quantum criptography, where after a necessary theoretical introduction showed a software able to simulate on three computers a practical implementation of a quantum criptography method; then he described the problem of teleportation illustrating the detailed analysis for an N-level quantum system due to L. A.. himself and M. Ohya and finally he showed the power of quantum computation in solving difficult problems, like the satisfyability problem, and discussed the Grover and Ohya-Masuda algorithms.

Paolo Gibilisco gave two talks on information geometry, the theory of statistical manifold, first with an overview of classical results and then with the advances on Fisher information and its geometry in a quantum context.

Matteo Paris gave two talks on the concept of entanglement in quantum mechanics, first overviewing its general properties and related concepts, then showing how one can use entangled states to reconstruct the action of a quantum system or to estimate some of its parameters.

On the subject of control and information science, **Fabio Fagnani** gave a series of four lectures on control problems with communication constraints. He focused on the problem of stabilysing a discrete time linear system by means of quantized controls (taking only a finite number of values). He described the new features of the problem with respect to classical stabilization theory and introduced quantities that measure the performance and the complexity of the stabilizers, namely the entrance time in an invariant region, a constant measuring the contraction and the number of control values. He presented several examples and then established, using symbolic dynamics and the ergodic theory of piecewise affine maps, general trade-off results between performance and complexity.

Finally, **Anant Sahai** gave a talk on information theory for non stationary inputs, a new subject that required to introduce non-trivial analogs of many fundamental concepts and results of classical information theory.

The third week addressed the many new and exciting connections between problems and algorithms in statistical physics, combinatorial optimization problems in computer science, and error correcting codes in information theory. This week was chaired by **Sekhar Tatikonda**. Because many of the participants have different backgrounds the week opened with a few review talks. In particular **David Forney** presented a review of modern coding theory and **Nicolas Sourlas** presented a review of the statistical mechanics of error correcting codes.

Techniques developed in the statistical mechanics community, such as the cavity method and the replica method, have been applied to combinatorial optimization problems with remarkable results. **Giorgio Parisi** discussed techniques for analyzing the chromatic number of random graphs. He first presented a random graph model with finite average connectivity and showed that the probability of short cycles occurring in the graph is small. He then discussed the importance of understanding the structure of the space of solutions which in this case is the set of satisfying colorings. In the situations where there are separate sets of solutions, so called metastable states, he presented a "whitening" algorithm that can be used to find a valid coloring. He ended with a discussion of how to determine the critical parameters of the random graph ensemble.

Andrea Montanari discussed the importance of understanding the proliferation of pure states in coding problems and satisfiability problems. He compared and contrasted the behavior of the sum-product decoding algorithm for LDPC codes and the survey propagation algorithm for the k-satisfiability problem. He then described the structure of the solution space for both problems and presented techniques for estimating the number of different solutions for different energy levels.

Riccardo Zecchina presented the survey propagation technique, a technique developed for dealing with the proliferation of metastable states in combinatorial optimization problems. He gave a detailed description of the algorithm and its application to the k-satisfiability problem. He then gave a demonstration of his software.

Rudiger Urbanke presented a complete story for the design of error correcting codes with the iterative decoding algorithm for the erasure channel. He first presented the underlying factor graph model and then discussed the importance of irregular graphs for achieving the Shannon capacity limit. He then discussed the density evolution analysis technique and EXIT charts.

Sekhar Tatikonda discussed the convergence of the sum-product algorithm for generic graphs. He showed how multiple Gibbs states, and hence multiple solutions, may arise. He presented conditions that insured convergence of the algorithm as well as bounds on the approximation error.

Pascal Vontobel discussed connections between the sum-product algorithm on factor graphs and electrical circuit theory. He presented interesting conditions and insights about the behavior of the sum-product algorithm for Gaussian distributions.

There were also talks on other connections between statistical mechanics and information theory. **Haggai Kfir** discussed some of the statistical mechanics aspects of the joint source channel coding problem. **Nicola Elia** presented work on the connections between optimal control problems with uncertainties and different channel coding problems with feedback. **Balaji Prabhakar** presented work on the limiting properties of large queuing systems.

Chandra Nair presented a technique for analyzing the finite assignment problem. He showed the correctness of the Parisi and Coppersmith-Sorkin conjectures.

The fourth week addressed topics of information theory, nonequilibrium statistical mechanics and disordered systems.

Andrea Montanari gave two tutorial talks on probabilistic models on graphs with particular emphasis on the problem of calculating marginals of a given probability distribution. This problem arises naturally in many different contexts like coding theory, Bayesian models and statistical mechanics. In the first talk he recalled the concept of Tanner graph, already seen in the previous week and introduced the probabilistic structure on such graphs. With the aid of several examples he showed how this structure can be found in many different scientific areas already mentioned above. He then introduced the belief propagation algorithm which allows the computation of the marginals in an exact way in the case when the graph is actually a tree. In the second talk he furnished a physical interpretation of the belief propagation algorithm as the Bethe energy minimization problem. He finally gave physical insight on the reason why the algorithm still works for loopy Tanner graphs without short cycles.

A number af lectures have been devoted to spin glass models, in particular the Sherrington-Kirkpatrick model. **Francesco Guerra** gave two lectures on his recent advances (partially joint works with F.L. Toninelli) on this problem. He recalled Parisi formula for the free energy in terms of the solution to an Hamilton-Jacobi equation (a formula proved only heuristically) and proved rigorously that the annealed free energy is well defined and takes a value less or equal to the Parisi solution, giving in this way also a clear motivation for the emergence of all the terms of that formula (at least as a rigorous upper bound). He jointly used cavity methods and some special interpolation methods, both to prove that the annealed free energy exists and to bound it by the Parisi solution (two different interpolations have been described). Finally he highlighted the very recent advance of Aizenmann-Sirius-Star and its relations with the previous ideas and results.

Fabio Lucio Toninelli completed this picture with a lecture devoted to the high temperature regime, above the Almeida-Thouless line, where he and Guerra proved that Parisi solution is correct. For zero external field the proof is simply based on Jensen inequality and a second moment method, but for positive h a suitable interpolation is used.

Pierluigi Contucci gave another lecture on the subject, mostly devoted to the Edwards-Anderson model but also to a more general context, where he showed the use of interpolation ideas together with Griffiths' inequalities.

Finally, **Marco Isopi** gave a lecture on the dynamic of a spin glass model, a random walk on the one-dimensional lattice with long tail holding times. He described a scaling limit where a singular diffusion arises and gave results of aging and localization. This lecture emphasized also the relation with the spin glass models treated in the first week by Ben Arous.

Problems, results and perspectives in nonequilibrium Statistical Mechanics have been the subject of another series of talks. **Giovanni Jona-Lasinio** discussed how to modify, for nonequilibrium stationary states (nonreversible processes), Onsager symmetry relations and Onsager-Machlup characterization of the most probable trajectory which gives us the fluctuation. The principle of minimal dissipation of Onsager has been also discussed and a formulation of nonreversible processes for certain interacting particle systems was given. Finally, the trajectory that produces the fluctuation has been interpreted in the framework of optimal control theory: for a suitable choice of the cost functional, the optimal control corresponds to the external field that produces the fluctuation.

Lorenzo Bertini discussed the general theory presented by Jona Lasinio with more mathematical details in the concrete example of the boundary driven simple exclusion process. He discussed the probabilities of large deviations with respect to the hydrodynamic equations and then showed that the computation of the quasi-potential can be performed by a nonlinear boundary value problem.

The fifth week was devoted to percolation and related models.

Alberto Gandolfi overviewed Discrete and Continuum percolation. In the first lecture he defined the problem and outlined the proofs of some estimates of the critical probability pc of percolation. In the second lecture he concentrated on the discrete problem: he studied the number of clusters and the probability pp that the origin is part of an infinite cluster: he proved that this probability is continuous when p> pc, the critical probability; he outlined the differences in proving these same results for the continuum case; in the end he started explaining rescaling methods and the tiling approach for the 2-dimensional discrete problem. In the last lecture he continued the study of the discrete 2-dimensional problem; by tiling and rescaling he proved that the probability pp is continuous, and that for p= pc there is almost certainly no infinite cluster of either open or closed points (that is, there is no percolation or anti-percolation).

Rauhl Roy addresses Random directed spanning trees. In the first lecture he defined 3 kinds of spanning trees (Euclidean minimal spanning trees, uniform ones, directed ones) and addressed the questions of the geometry of the graphs and the asymptotics of the finite space constructions. After a careful definition of the formal structure, he proved some results that show that, in 2 and 3 dimensions, the graph is almost certainly an infinite tree, while for dimension greater than 3 the graph is a.c. composed of infinite trees (a forest); moreover, for any dimension, these trees do not contain bi-infinite paths. Central limit theorems for the number of vertices with a given degree and for the number of edges of a fixed length have been also given. Dimension 2 is treated by a martingale argument; a Lyapunov technique in dimension 3 is described to study a recurrency property.

The sixth week was devoted to scaling limits and organized by **Paolo Dai Pra**, **Marco Isopi** e **Daniela Morato**.

The main theorems of probability theory, law of large numbers, central limit theorem, invariance principle, concern the study of rescaled sums. The study of the limiting behavior of rescaling procedures lies at the heart of such topics as fractals, renormalization group, as well scale invariant objects in pure and applied mathematics The special week on Scaling Limits aimed at introducing young researchers to some hot topics that are receiving a lot of attention for their own interest, as well as for the connection with many areas in mathematics and theoretical physics.

The lectures by **Fraydoun Rezakhanlou** were devoted to scaling limits for interacting particle systems. The problem of deriving, starting from Hamiltonian dynamics, the equations that describe macrosopic behavior of large systems has been at the centerstage of science since the work of Boltzmann. The lectures started with some classical open problem in this context. Depending on the way the number of particles is scaled with the range of interaction, one expect to have kinetic, hydrodynamic or diffusive behavior. Due to the existence of several conserved quantities the derivation of macroscopic equations is beyond the reach of present day techniques.

More can be done in the scalar case by considering some stochastic variations of the

Hamiltonian systems so that only one quantity is conserved and some of the expected macroscopic behavior can be rigorously established.

The lectures by **Boris Tsirelson**, *Scaling limits: from unstable random walks to nonclassical Brownian motions*, started by describing a general framework for talking about scaling limits without relying on topological notions. Some meaningful examples were discussed, first linear functions of many independent random variables which lead to classical noises. Afterwards two examples were examined, the "noise made by a Poisson snake" (Warren) and the "Brownian web as a black noise" (Tsirelson). Those involve very nonlinear functions and lead to nonclassical noises.

In his lectures, *Conformal Fractal Geometry and Quantum Gravity*, **Bertrand Duplantier** considered the general class of conformally invariant critical random curves in the plane. The multifractal distribution for the scaling and winding of the electrostatic potential lines near their boundary has been derived from two-dimensional quantum gravity techniques. This has been applied to the O(N) and Potts models, as well as to the SLE (Stochastic Loewner Evolution).

Ronald Meester continued the subject of the previous week and described a variant of continuum percolation which has a direct interpretation for networks. Given Poisson distributed clients, he considered covering algorithms by balls, such that all balls cover at least one client and all clients are covered. Percolation of the coverings have been analyzed, under various conditions for the coverings (flat, shift invariant, and others) and results on critical exponents have been given.

The last (seventh) week was mainly devoted to network information theory.

P.R. Kumar gave three talks on this subject, central in spirit to the main purposes of the full period. He addressed the question of the capacity of wireless networks and considered the so called protocol model of interference (and a generalization to the physical model) with n nodes in a region of given area and asked questions like the throughput for each node, the capacity of the entire network and the scaling properties with n. First he considered the best possible scenario, with optimal spatial and temporal scheduling and routes and gave asymptotic with n lower and upper bounds for the capacity (both square root of n). Then he gave the analogous results for a random location model, where now the scaling is worse by a factor log n. The strategy in these models is multi hop, without attempts to take advantage of the interference. The proofs of the upper bounds are based on book-keeping of certain constraints, while the proof of the lower bounds, easy in the best possible scenario, is non trivial in the random location case, with use of techniques like Voronoj tassellations and uniform law of large numbers. The subject was extended in various direction and several related items have been discussed, like protocol design, architectures, power control problems, media access control, routing tables and algorithms.

Balaji Prabhakar gave two talks. The first one was devoted to simple, scalable network algorithm. He introduced several concepts, methods and problems, like randomization, throughput, eviction schemes (deterministic and randomized), switch scheduling, incentive mechanisms for diffusing focused load, and the Choke algorithm. The second talk was devoted to queuing and network queuing theory. Basic questions were the preservation of Poisson property by certain queues and the convergence to Poisson process of a number of schemes and transformations, so that Poisson is seen as a fixed point. Entropy methods and reversibility are two of the main ingredients to analyze these questions. Several models have been addresses, like first in first out, last in first out, and others, and the East model.

Nigel Newton gave two talks on the connections between estimations problems and statistical mechanics. He introduced a variational principle (similar to the free energy principle) for a very general class of Bayes conditional distributions. An energy is associated to such measures and the principle involves entropy and information quantities associated to the energy and certain measures. As an application, he discussed a nonlinear filtering problem and its relation with a stochastic optimal control problem (related to the variational principle). He also described concepts and results about the flow of information and the production of entropy in sequential estimators.

Massimo Franceschetti gave a talk on the importance of the concept of phase transition in engineering problems. Among the various examples, he also gave additional results on the phase transition of the model of covering Poisson distributed clients, previously presented by Meester.

Related to the activities of the period, the **Cattedra Galileiana 2003** (supported by the Association *Amici della Scuola Normale Superiore di Pisa*) was covered by S. R. Varadhan, who gave a series of 16 one-hour lectures on *Large deviations and Exclusion Processes*. Here is a rough summary of the extensive material treated in his talks:

basic facts on particle systems, invariant measures
hydrodynamic limit of the symmetric simple exclusion processes
large deviation estimates, Dirichlet forms and Feynman-Kac formula
one-block and two-block estimates
non-gradient systems, simple exclusion with more than one color,
their hydrodynamic limits, large deviations; self-diffusion coefficient
large deviations of the empirical process
totally asymmetric simple exclusion and its hydrodynamic limit
entropy solutions to the Burgers equation
entropy tracking, microscopic and macroscopic entropies
exponential and super exponential estimates
weak solutions with non-entropic shocks, their large deviation
functional limiting density flux by slowing down the process.

The material of the lectures will presumably be organized in a research monograph; preliminary material has been distributed during the lectures.

Besides the speakers and organizers mentioned above, the Centro received as visitors several people, some of them contributing also with short talks, and some of them receiving granted supports for their visits.

They include: I. Emre Telatar, Vincenzo Capasso, , Maria Simonetta Bernabei, Claudio Altafini, Giacomo Aletti Ida Minelli, Mattia Fedrigo, Cyrill Measson , Bruno Picasso, Francesco Prodi, Massimiliano Gubinelli, Chandrasekar Madhavan-Nair,Hakima Bessaih, Justin Dauwels, Giovanni Sebastiani, Roberto Guenzani, Changyan Di, Davide Gabrielli, Cristina Toninelli, Anne-Séverine Boudou, Simone Gambini, Emiliano Casalicchio, Gautam Gupta , Vishwambhar Rathi, Francesco Morandin, Giuseppe Nicosia, Francesco Caravenna, Yuhong LI, Ashish Khisti, Shashibhushan Borade, Shan-Yuan Ho.