ERC conference on Optimal Transportation and Applications
Centro De Giorgi, Scuola Normale Superiore
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Titles and abstracts

Yann Brenier (École Polytechnique). *Optimal transport and combinatorial optimization: old and new.*
Abstract. There are well-established connections between combinatorial optimization, optimal transport theory and Hydrodynamics, through the linear assignment problem in combinatorics, the Monge-Kantorovich problem in optimal transport theory and the model of inviscid, potential, pressure-less fluids in Hydrodynamics. Here, we consider the more challenging quadratic assignment problem (which is NP, while the linear assignment problem is just P) and find, in some particular case, a correspondence with the problem of finding stationary solutions of Euler's equations for incompressible fluids.
Ref. ArXiv:1410.0333 .

José Antonio Carrillo (Imperial College) *Minimizers of Interaction Energies.*
Abstract. I will start by reviewing some recent results on qualitative properties of local minimizers of the interaction energie to motivate the main topic of my talk: to discuss global minimizers. We will show the existence of compactly supported global miminizers under quite mild assumptions on the potential in the complementary set of classical H-stability in statistical mechanics. A strong connection with the classical obstacle problem appears very useful when the singularity is strong enough at zero. An approach from discrete to continuum is also quite nice under convexity assumptions on the potential.
This is based on three preprints/works in preparation one together with F. Patacchini, J.A. Caizo, another one with M. Delgadino and A. Mellet, and finally with M. Chipot and Y. Huang.

Matthias Erbar (Bonn University). *Curvature-dimension bounds for continuous and discrete spaces.*
Abstract. Metric measure spaces with curvature-dimension bounds in the sense of Lott-Villani-Sturm are known to satisfy are number of strong properties in terms of geometric and functional inequalities. In this talk we first present a new and more simple curvature-dimension condition based solely on convexity properties of the Shannon entropy. In combination with linearity of the heat flow it allows to establish a Bochner formula as the starting point for a much finer analysis on metric measure spaces. Then we make the connection to the setting of discrete spaces and Markov chains where a notion of lower curvature bound in the spirit of Lott-Villani-Sturm has been developed recently. The same new curvature-dimension condition comes as a natural reenforcement of this notion and allows to give a new notion of dimension for discrete spaces. We will discuss first

consequences of this notion as well as new results on Ricci bounds for discrete spaces. Joint work with Kazumasa Kuwada, Theo Sturm and work in progress with Chris Henderson, Jan Maas, Georg Menz and Prasad Tetali.

Wilfrid Gangbo (Georgia Institute of Technology). *Metric Viscosity Solutions of Hamilton-Jacobi Equations Depending on Local Slopes.*

Abstract. We present a theory of metric viscosity solutions which encompasses a large class of Hamiltonians. We consider time dependent problems

$$\partial_t u + H(t, x, u, |\nabla u|) = 0, \quad \text{in } (0, T) \times \Omega, \tag{1}$$

$$\begin{cases} u(t, x) = f(t, x) & \text{on } (0, T) \times \partial\Omega, \\ u(0, x) = g(x) & \text{on } \Omega, \end{cases} \tag{2}$$

and stationary equations. We prove a range of comparison and existence results that apply to a wide range of equations and we present a sample of techniques that can apply in other cases. (This talk is based on a joint work with A. Swiech).

Nicola Gigli (UPMC, Jussieu). *Nonsmooth differential geometry.*

Abstract. I shall describe in which sense every metric measure space possesses a first order differential structure and how a second order one arises in presence of a lower Ricci curvature bound.

Nathael Gozlan (Université Paris Est Marne la Vallée). *Transport proof of weighted Poincaré inequalities for log-concave probability measures.*

Abstract. We prove, using optimal transport tools, weighted Poincar inequalities for log-concave random vectors satisfying some centering conditions. We recover by this way similar results by Klartag and Barthe-Cordero-Erausquin for log-concave random vectors with symmetries. In addition, we prove that the variance conjecture is true for increments of log-concave martingales. Joint work with D. Cordero-Erausquin.

Martin Huesmann (Bonn University). *Optimal transport and Skorokhod embedding.*

Abstract. The Skorokhod embedding problem is to represent a given probability as the distribution of Brownian motion at a chosen stopping time. Over the last 50 years this has become one of the important classical problems in probability theory and a number of authors have constructed solutions with particular optimality properties. These constructions employ a variety of techniques ranging from excursion theory to potential and PDE theory and have been used in many different branches of pure and applied probability.

We develop a new approach to Skorokhod embedding based on ideas and concepts from optimal mass transport. In analogy to the celebrated article of Gangbo and McCann on the geometry of optimal transport, we establish a geometric characterization of Skorokhod embeddings with desired optimality properties. This leads to a systematic method to construct optimal embeddings. It allows us, for the first time, to derive all known optimal Skorokhod embeddings as special cases of one unified construction and leads to a variety

of new embeddings. While previous constructions typically used particular properties of Brownian motion, our approach applies to all sufficiently regular Markov processes.
This is joint work with Mathias Beiglbck and Alexander Cox.

Christian Ketterer (Bonn University). *Obata's theorem for metric measure spaces.*
Abstract. In this talk I will present a version of Obata's rigidity theorem for metric measure spaces that satisfy a Riemannian curvature-dimension condition. More precisely, if there is equality in the Lichnerowicz eigenvalue estimate, the space is a spherical suspension.

Kazumasa Kuwada (Ochanomizu University). *On the speed in Transportation costs of heat distributions.*
Abstract. By regarding heat distributions as a curve in the space of probability measures, we can consider its speed measured by some transportation costs. Our main concern in this talk is heat distributions on (backward) Ricci flow. The speed can be expressed explicitly when heat distribution is identified with a gradient flow of the relative entropy. On Ricci flow, this interpretation does not seem to work well. Nevertheless we can show some results for a suitably chosen transportation costs. By combining this result with monotonicity of those transportation costs, we can show the monotonicity of Perelmans F-functional as well as W-entropy. Indeed it extends some known results on Ricci flow to noncompact case.

Christian Leonard (Université Paris Ouest Nanterre). *An entropic interpolation problem for incompressible viscid fluids.*
Abstract. In 1966, Arnold proposed to look at Euler's equation for perfect fluids as describing the geodesic flow of volume preserving diffeomorphisms. In the same spirit, in 1989 Brenier designed a least action principle based on optimal quadratic transport which allows for getting rid of the high regularity assumptions which underly Arnold's approach. Replacing deterministic geodesics on the state space by sample paths of Brownian bridges and optimal transport by minimal entropy, we obtain a least action principle for the Navier-Stokes equation, very much in the spirit of Brenier's representation of the Euler equation. This is a joint work with M. Arnaudon, A.-B. Cruzeiro and J.-C. Zambrini.

Matthias Liero (WIAS, Berlin). *Reaction-diffusion via optimal transport on the cone space.*
Abstract. It was recently shown by Mielke that a wide class of reaction-diffusion systems can be formulated in a natural way via gradient structures for the relative entropy or free energy. The metric gradient of the driving functional is determined via a state-dependent Onsager operator containing a diffusion part of Wasserstein type and an additional reaction term. With the Onsager operator we can associate a dissipation distance in the sense of Benamou-Brenier by infimizing the total dissipation over all connecting curves. The question of attainment of this infimum, which is the same as the existence of geodesic curves, is an open question in most cases. In this talk we present a full characterization

of a dissipation distance induced by a reaction-diffusion Onsager operator that depends linearly on the state. In particular, we show that the distance is given by the Kantorovich-Wasserstein distance on an extended space, which is given by the cone construction over the underlying domain. This is joint work with Alexander Mielke and Giuseppe Savar.

Jan Maas (Bonn University). *A gradient flow approach to chemical master equations.*
Abstract. Chemical reaction networks are often modelled stochastically as continuous time Markov chains. In this talk we present a gradient flow approach to the associated chemical master equation. In particular, we discuss convergence of the gradient flow structure in the thermodynamic limit, and analyse geodesic convexity properties of the relative entropy functional. This is joint work with Alexander Mielke.

Bertrand Maury (Université Paris Sud). *Crowd motion and pressureless Euler equation: an OT standpoint.*
Abstract. Accounting for congestion in microscopic crowd motion modeling leads to non-smooth evolution problems, which fit in the framework of sweeping processes introduced by J.J. Moreau in the 70'. A similar model can be written at the macroscopic level: a density of people is advected by a velocity field that is defined as the projection of some desired velocity field (the velocity that people would take without congestion) on the cone of feasible velocities (i.e. velocities that do not lead to a violation of the congestion constraint). The sweeping structure of this macroscopic problem can be recovered by endowing the set of densities with the Wasserstein metric, making it possible to obtain existence results and reliable numerical schemes. The second order in time counterpart of the microscopic model is the standard granular flow problem with non-elastic collisions. In spite of strange nonuniqueness issues, this model is essentially well-understood. It is then tempting to investigating the second order counterpart of the macroscopic model, that is the so-called pressureless Euler equation with congestion. It has been studied in the one-dimensionnal setting ("sticky blocks"), but the problem remains essentially open in higher dimensions. We shall present the links between those problems, and propose some exploratory leads to apply the Wasserstein setting to pressureless Euler equations with congestion. This work results from collaborations with J. Venel, A. Preux, A. Roudneff-Chupin, and F. Santambrogio.

Robert Mc Cann (Toronto University). *The spectrum of a family of fourth-order nonlinear diffusions near the global attractor.*
Abstract. The thin-film and quantum drift-diffusion equations belong to a fourth-order family of evolution equations proposed by Denzler and myself as analogous to the (second-order) porous medium family. They are 2-Wasserstein gradient flows of the generalized Fisher information (just as Otto showed the porous medium to be the 2-Wasserstein gradient flow of the Reyni entropy). In this talk we describe the linearization of the fourth-order dynamics around the self-similar solution. We diagonalize this linearization by relating it to analogous problem for the porous medium equation. This yields information

about the leading- and higher-order asymptotics of the fourth-order flows on $\mathbf{R}^n$ which — outside of special cases — were inaccessible previously. These results were obtained jointly with Christian Seis.

Facundo Memoli (Ohio State University). *The shape space defined by the Gromov-Wasserstein distance.*
Abstract. In a number of applications, datasets or shapes can be modeled as metric measure spaces. The Gromov-Wasserstein distance –a variant of the Gromov-Hausdorff distance based on ideas from mass transport– provides an intrinsic metric on the collection of all mm-spaces. I will review its construction, main properties, lower bounds, and computation.

Alpar Meszaros (Université Paris Sud). *A diffusive crowd motion model with density constraints.*
Abstract. In this work we present a macroscopic crowd motion model under hard congestion effects with a non-degenerate diffusion in the movement. This could be seen as a second order version of the models studied by B. Maury, A. Roudneff-Chupin and F. Santambrogio (2010). In our model we consider general velocity fields (not necessarily gradients, and with minimal regularity assumptions) and study splitting-type schemes to show that the problem has a solution. The scheme is constructed with the help of the Fokker-Planck equation and the projection operator in the Wasserstein space. Compactness estimates to prove convergence are obtained by standard comparisons between metric derivative in W_2 and dissipation of the entropy, together with the analysis of the projection operator. This operator decreases both the entropy and the total variation. This also allows to provide Lipschitz in time estimates for the curves of densities of the population w.r.t. the 1-Wasserstein distance. This is a joint work with F. Santambrogio.

Alexander Mielke (WIAS, Berlin). *A reaction-diffusion equation as a Hellinger-Kantorovich gradient flow.*
Abstract. Large classes of reaction-diffusion systems with reactions satisfying mass-action kinetics and the detailed-balance condition can be written as a formal gradient system with respect to the relative entropy. The the dual dissipation potential is the sum of a transport part for diffusion and a reaction part. We discuss the mathematical steps needed to turn the formal theory into a rigorous metric gradient system.
Motivated by scalar reaction-diffusion equations we construct the so-called Hellinger-Kantorovich distance on the set of all non-negative measures. This distance can be obtained (i) via transport and growth, (ii) by the inf-convolution of the Kantorovich-Wasserstein distance and the Hellinger distance, and (iii) by minimizing a logarithmic-entropy transport problem. We provide examples of entropies and such that induced reaction-diffusion equation is a lambda-convex gradient flow.
This is joint work with Matthias Liero (WIAS Berlin) and Giuseppe Savare (Pavia).

Andrea Mondino (ETH, Zurich). *On the local structure of $RCD^*(K,N)$-spaces.*
Abstract. $RCD^*(K,N)$-spaces are finite dimensional metric measure spaces satisfying a synthetic lower Ricci curvature bound and having linear Heat flow. In the seminar we will discuss some recent progresses about how these two conditions imply some local euclidean structure.

Mark Peletier (Eindhoven University). *Stochastic origins of gradient flows: a general connection.*
Abstract. Although it has been known for some time that certain gradient-flow structures are related to the large deviations of a stochastic process, until recently we only understood this at the level of examples. In this lecture I will explain a general structure that gives rise to the following property: for every sequence of reversible stochastic processes with a large-deviations principle, the limiting equation is a generalized gradient flow that maps one-to-one to the large-deviations rate function. Therefore the large class of reversible stochastic processes generates a correspondingly large class of generalized gradient flows. This is joint work with Michiel Renger and Alexander Mielke.

Andrea Pinamonti (Scuola Normale Superiore). *Tensorization of Cheeger energies and applications to the area formula for graphs in metric measure spaces.*
Abstract. We study the tensorization properties of weak gradients in metric measure spaces (X, d, m). We apply these results to compare the area functional with the perimeter of the subgraph, in the same spirit as the classical theory.

Shin-ichi Ohta (Kyoto University). *Gradient flows of semi-convex functions on CAT(1)-spaces.*
Abstract. We generalize the theory of gradient flows of semi-convex functions on CAT(0)-spaces, by Mayer and Ambrosio-Gigli-Savare, to CAT(1)-spaces. The key tool is the so-called "commutativity" following from the first variation formula including the angle. The commutativity enables us to use the semi-convexity of the distance function and the Riemannian nature of the space separately. Joint work with Miklos Palfia (Kyoto University).

Tapio Rajala (Jyväskylä University). *Examples of branching metric spaces with Ricci curvature lower bounds.*
Abstract. In this talk I will discuss the role of the non-branching assumption in MCP(K,N) and CD(K,N) spaces via examples that are subsets of the plane with the supremum norm. The focus will be on the local-to-global property, splitting and maximal diameter theorems, dimensions, tangents and existence of optimal transport maps. Part of the results are joint work with Christian Ketterer.

Filippo Santambrogio (Université Paris-Sud). *On the solutions of the variational problem $\min_\rho W_2^2(\rho, \nu) + F(\rho)$: new estimates and applications.*
Abstract. The problem of minimizing the sum of the squared Wasserstein distance to a given measure plus a penalization appears very often in the applications of transport

theory: it appears in the time-discretization of gradient flows or other evolution equations, in regularization procedures in image processing, in spatial economics... I will review the optimality conditions for this problem and the consequences that they have in terms of estimates on the optimal ?. In particular I will concentrate on L^∞ and BV estimates in the case where F is the integral of a convex function $f(?(x))$. The BV case has been recently obtained with G. De Philippis, A. Mészáros and B. Velichkov, and has interesting applications to the problem of the Wasserstein projection onto the set of densities bounded by a given constant, that we constantly used with B. Maury in the optimal transport approach to crowd motion. The L^∞ case is more classical and mainly based on the use of the Monge-Ampre equation. Yet, a similar strategy can also be applied when F is the squared H^{-1} norm (given by a logarithmic interaction term in dimension 2). This allows to deal with the parabolic-elliptic Keller-Segel equation and obtain very general L^∞ bounds for small time, under any type of diffusion, and for supercritical or subcritical mass. This part of the talk, where I will only sketch the ideas of the estimate, comes from a work-in-progress with J.-A. Carrillo.

Takashi Shioia (Tohoku University). *Metric measure limits of spheres and complex projective spaces.*

Abstract. I will talk on the study of the limits of sequences of spheres and complex projective spaces with unbounded dimensions. A sequence of spheres (resp. complex projective spaces) either is a Levy family, infinitely dissipates, or converges to (resp. the Hopf quotient of) a virtual infinite-dimensional Gaussian space, depending on the scale of the spaces. These are the first discovered examples with the property that the limits are drastically different from the spaces in the sequence. For the proof, we define a metric on Gromovs compactification of the space of metric measure spaces.

Eugene Stepanov (St. Petersburg University). *Flows of measures induced by measurable vector fields.*

Abstract. A smooth vector field (say, over a mainfold) may be defined either as a linear operator on the algebra of smooth functions satisfying Leibniz rule, or, equivalently, as a smooth field of directions of curves (i.e. "vectors") at every point. The first notion easily generalizes to what is known as measurable vector fields introduced by N. Weaver. These vector fileds can in fact be identified with one-dimensional metric currents of Ambrosio and Kirchheim. We show that also the second identification (with rectifiable curves in place of smooth ones) is valid for a large class of measurable vector fileds (but not all of them) and study the analogues of integral curves and ODE's produced by such vector fields as well as the flows of measures generated by them.

Karl-Theodor Sturm (Bonn University). *Recent results in analysis on metric measure spaces.* **Abstract.** We present some recent developments in the geometric analysis on mms and in the study of diffusion operators. Particular topics will be:

* gradient flows for semiconvex functions on mms;
* self-improvement of Bochner's inequality and applications to time changes and conformal transformations;
* convexification of domains in mms and analysis of Neumann Laplacians.

Dario Trevisan (Scuola Normale Superiore). *DiPerna-Lions flows in* $\mathsf{RCD}(K,\infty)$ *metric measure spaces.*

Abstract. This talk is based on a recent joint work with L. Ambrosio, where a DiPerna-Lions theory of flows of ODEs associated to Sobolev vector fields is established, in a rather general setting. Our aim is to highlight the role played by an abstract superposition principle, linking "Eulerian" (continuity equations) and "Lagrangian" (flows of ODEs) viewpoints, and by curvature assumptions on the underlying geometry, providing satisfactory theories, together with non-trivial examples of flows, in the class of $\mathsf{RCD}(K,\infty)$ metric measure spaces.