INdAM Intensive research period
Perspectives in Lie Theory

Abstracts

Session 2: Winter School: Lie Theory and Representation Theory

This session is organized in the framework of the Summer and Winter Schools of the University of Padova.

Period: January 19 – February 6, 2015

Mini-courses

Alexander Premet
Finite W-algebras, primitive ideals and modular representations

Let be G a simple algebraic group over $\mathbb{C}$ with Lie algebra $\mathfrak{g}$ and let e be a nilpotent element of $\mathfrak{g}$. The aim of this lecture course will be the interplay between finite dimensional irreducible representations of the finite W-algebra $U(\mathfrak{g}, e)$ and primitive ideals of the universal enveloping algebra $U(\mathfrak{g})$ whose associate variety coincides with the Zariski closure of the adjoint G-orbit of e . Some applications to the representation theory of the modular counterpart of $\mathfrak{g}$ will also be discussed.

Vera Serganova
Representations of Lie superalgebras

The goal of this minicourse is to review recent results in representation theory of finite-dimensional Lie superalgebras.

- Lecture 1 Finite-dimensional Lie superalgebras: classification, examples, highest weight theory, first results about characters.

- Lecture 2 Center of universal enveloping superalgebras. Blocks in the category of

  finite-dimensional representations and category $\mathcal{O}$ Stein-Gelfand-Gelfand reciprocity. Weight diagrams.

- Lecture 3 Algebraic supergroups. Geometric methods: Borel-Weil-Bott theory, associated variety and odd nilpotent cone.

- Lecture 4 Schur-Weyl duality for different classical superalgebras. Deligne's theorem and relation with tensor categories.

- Lecture 5 Categorification of Fock space via translation functors for superalgebras. Kazhdan-Lusztig-Brundan conjecture for category $\mathcal{O}$ in supercase.

Geordie Williamson
An illustrated guide to perverse sheaves

This mini-course will be an introduction to perverse sheaves, with emphasis on examples from representation theory. It will be a course full of pictures and examples, with the aim of trying to get some feeling for the fundamentals of perverse sheaves: t-structures and gluing, intersection cohomology complexes, the decomposition theorem, vanishing and nearby cycles. As a grand finale I hope to cover de Cataldo and Migliorini's proof of the decomposition theorem. Although the most important concepts will be recalled, I will assume a basic knowledge of sheaves and derived categories. (If you have never worked with sheaves, cohomology or derived categories before the learning curve will be steep.)

Research Talks

MARIA CHLOUVERAKI *The Yokonuma-Hecke algebra of type A*

Yokonuma-Hecke algebras were introduced by Yokonuma in the 60's as generalisations of Iwahori-Hecke algebras. They have recently attracted the interest of topologists, because they naturally give rise to invariants for framed and classical knots. In this talk we will introduce and study the Yokonuma-Hecke algebra of type A from both algebraic and topological points of view.

STEPHANE GAUSSENT *Hecke algebras associated to Kac-Moody groups over local fields*

Spherical or Iwahori-Hecke algebras associated to a reductive group over a local field are well known and have a lot of applications in representation theory and in arithmetics. Braverman, Kazhdan and others have extended their construction to the case of affine Kac-Moody groups. In the talk, we will construct these algebras for any Kac-Moody group over a local field using the hovel (an adapted version of the Bruhat-Tits building). This is a joint work with Nicole Bardy-Panse and Guy Rousseau. /tmp/kde-thuillier/okularR29630.tmp

JING-SONG HUANG *Dirac operators in representation theory*

We plan to begin with a retrospective of the recent development of Dirac cohomology from Vogan's conjecture and its various generalizations and extensions. Then we discuss how to make use of the Dirac cohomology for calculation the characters of elliptic representations and endoscopic transfer factors.

PAUL LEVY *Dual singularities in exceptional type nilpotent cones*

It is well-known that nilpotent orbits in $\mathfrak{sl}_n(\mathbb{C})$ correspond bijectively with the set of partitions of n, such that the closure (partial) ordering on orbits is sent to the dominance order on partitions. Taking dual partitions simply turns this poset upside down, so in type A there is an order-reversing involution on the poset of nilpotent orbits. More generally, if $\mathfrak{g}$ is any simple Lie algebra over $\mathbb{C}$ then Lusztig-Spaltenstein duality is an order-reversing bijection from the set of special nilpotent orbits in $\mathfrak{g}$ to the set of special nilpotent orbits in the Langlands dual Lie algebra $\mathfrak{g}^L$. It was observed by Kraft and Procesi that the duality in type A is manifested in the geometry of the nullcone. In particular, if two orbits $\mathcal{O}_1 < \mathcal{O}_2$ are adjacent in the partial order then so are their duals $\mathcal{O}_1^t > \mathcal{O}_2^t$, and the isolated singularity attached to the pair $(\mathcal{O}_1, \mathcal{O}_2)$ is dual to the singularity attached to $(\mathcal{O}_2^t, \mathcal{O}_1^t)$: a Kleinian singularity of type A_k is swapped with the minimal nilpotent orbit closure in $\mathfrak{sl}_{k+1}$ (and vice-versa). Subsequent work of Kraft-Procesi determined singularities associated to such pairs in the remaining classical Lie algebras, but did not specifically touch on duality for pairs of special orbits. In this talk, I will explain some recent joint research with Fu, Juteau and Sommers on singularities associated to pairs $\mathcal{O}_1 < \mathcal{O}_2$ of (special) orbits in exceptional Lie algebras. In particular, we (almost always) observe a generalized form of duality for such singularities in any simple Lie algebra.

NICOLAS LIBEDINSKY *Some questions about Soergel bimodules*

We will discuss some topics about Soergel bimodules related to the newly opened approach to the characters of irreducible rational representations of algebraic groups in positive characteristic and in particular a conjecture about the existence of an important morphism between Bott-Samelson bimodules that could approach us in some (yet to be discovered) way to these representations.

GUNTER MALLE *Decomposition numbers and Lusztig induction*

There does not (yet) exist a general theory for describing decomposition numbers of finite reductive groups at primes different from the defining characteristic. We explain a new conjecture predicting that characters of intersection cohomology complexes related to Lusztig induction should provide many new projective characters. We also present new, almost complete results on decomposition matrices of finite unitary groups of rank at most 10. This is joint work with Olivier Dudas.

GUIDO PEZZINI *Spherical subgroups of Kac-Moody groups*

We discuss the definition and properties of spherical subgroups of finite type of a Kac-Moody group. In analogy with the standard theory of spherical varieties, we introduce a combinatorial object associated with such a subgroup, its homogeneous spherical datum, which satisfies the same axioms as in the finite-dimensional case.

PENG SHAN *On the centre of quiver Hecke algebras*

In this talk, I will first explain a current algebra action on the center and cocenter of cyclotomic quiver Hecke algebras, and compare them to the cohomology of Nakajima quiver varieties. I will also explain in the case of Jordan quiver how to get a W-algebra action on the center of cyclotomic degenerate affine Hecke algebras and how to use it to compute the cohomology of Gieseker spaces. This is a joint work with M. Varagnolo and E. Vasserot.

RUDOLF TANGE *Highest weight vectors and transmutation*

I will consider the problem of finding good bases for the highest weight vectors for the polynomial functions on nxn matrices under the conjugation action in any characteristic. First we reduce this to finding good bases for the highest weight vectors for the polynomial functions on the nilpotent cone. Then we pass via "transmutation" to tuples of matrices. This leads to several interesting open problems, representation theoretic and combinatorial.

DONNA TESTERMAN *Distinguished unipotent elements and multiplicity free subgroups.*

Let G be a simple algebraic group of rank at least 2 defined over an algebraically closed field of characteristic 0 and let f be an irreducible representation of G whose image lies in SL(V), Sp(V) or SO(V). We consider two closely related problems, first the problem of determining those representations for which distinguished unipotent elements of G are sent to distinguished elements of the classical group on V. Also we settle a base case of the general problem of determining when the restriction of f to a simple subgroup of G is multiplicity-free. This is joint work with Martin Liebeck and Gary Seitz.

OKSANA YAKIMOVA *On symmetric invariants of semi-direct products.*

Let $\mathfrak{g}$ be a complex reductive Lie algebra. By the Chevalley restriction theorem, the subalgebra of symmetric invariants $S(\mathfrak{g})^{\mathfrak{g}}$ is a polynomial ring in rank $\mathfrak{g}$ variables. A quest for non-reductive Lie algebras with a similar property has recently become a trend in invariant theory. Several classes have been suggested, centralisers of nilpotent elements (Premet's conjecture), truncated bi-parabolic subalgebras (Joseph's conjecture), $\mathbb{Z}_2$-contractions (Panyushev's conjecture). We will discuss results on symmetric invariants of semi-direct products and their relevance to these conjectures.

Workshops

Title and Abstracts for the Super Quantum Lie day and the Cluster Algebra day will be given in the coming days.