Two weeks in global analysis - February 14-26, 2005

Global analysis is a broad field, involving functional analysis, differential geometry and algebraic topology. The aim of this school was to present an overview of the general methods of C^*-algebras, elliptic theory, degree theory and bifurcation, and to focus the attention to the Dirac operator in relativistic quantum mechanics, and to the the Cauchy-Riemann operator in symplectic geometry. The main activity consisted in six series of lectures, of 5 hours each. These lectures started from an introductory level, moving then to problems of current research interest and to open questions.

Jacobo Pejsachowicz: *Topological invariants in bifurcation theory of Fredholm maps.* Consider two Banach spaces X, Y and a family of maps $f : P \times X \to Y$ parametrized by a manifold P, such that $f(p, 0) = 0$ for every $p \in P$. A necessary condition for $(\overline{p}, 0)$ to be a bifurcation point is that the linearization $L(\overline{p}) = D_x f(\overline{p}, 0)$ is singular, and bifurcation theory calls for finding sufficient conditions. When the map L takes values into the space $\Phi_0(X, Y)$ of Fredholm operators of index zero - which has a rich topology - bifurcation is determined by the homotopy class of $L : P \to \Phi_0(X, Y)$. When the space of parameters P is one-dimensional, non-triviality of the homotopy class of L can be detected by a $\mathbb{Z}_2$ invariant, the parity, or in the variational case by a $\mathbb{Z}$-invariant, the spectral flow. When P is higher dimensional, the relevant invariants come from the K-theory of P. Indeed, the homotopy classes of maps from P to $\Phi_0(X, Y)$ are in one-to-one correspondence with stable equivalence classes of vector bundles over P.

Alexander Mishchenko: *C^*-algebras and elliptic theory.* The classical theory of vector bundles, characteristic classes, and K-theory were treated, in connection with the theory of differential and pseudo-differential operators on manifolds. Then its algebraic counterpart was outlined, including the theory of C^*-algebras, of Hilbert modules, operator K-theory and KK-theory, as well as Connes and Higson E-theory. The last lectures were devoted to applications to topology and geometry, including the Poincaré complexes and Pontrjagin duality, the Novikov conjecture, and the Baum-Connes conjecture.

Eric Serè and Maria Esteban: *Nonlinear models in relativistic quantum mechanics, I and II.* The Dirac operator is an unbounded self-adjoint operator, whose spectrum is the set of real numbers t with $|t| \geq 1$. Various Hardy-like inequalities allow to couple the Dirac operator with a Coulomb potential, providing a relativistic model for describing atoms and molecules. The effect of this potential is to create eigenvalues in the spectral gap $[-1, 1]$, whose precise values can be estimated by suitable generalizations of the Rayleigh quotients. When the study of the Dirac equation is too complicated, simplified models, such as the Dirac-Fock equation, bring in the study of nonlinear equations.

Matthias Schwarz: *On Floer homology and applications in Hamiltonian dynamics.* Floer homology was originally developed in order to prove the Arnold conjectures on the existence of fixed points of symplectic diffeomorphisms. However, it was soon realized to provide a powerful tool to build symplectic invariants: as a graded group Floer homology of a compact manifold M is nothing but the singular homology of M, but its ring structure is in general richer, encoding the quantum cap product. Also the Floer homology of the cotangent bundle of a compact manifold P is of particular interest, because its ring structure is the loop product on the homology of the loop space of P.

Octave Cornea: *Extended Morse-Floer complex in symplectic topology.* The Floer complex, as the Morse complex in classical Morse theory, only encodes information carried by one-dimensional moduli spaces. It is possible to encode also the information carried by higher dimensional moduli spaces, by enlarging the space of coefficients. This extended Floer complex comes with a differential and a grading, and the associated spectral sequence recovers the Serre spectral sequence of some suitable fibrations. This topological machinery can be applied to prove existence results for J-holomorphic curves.

The following talks by some of the younger participants were organized in the afternoons:

Giulio Minervini Finite volume flows and Morse dynamics.

Adriano Pisante A variational problem in $H^{1/2}(S^1, S^1)$ with lack of compactness.

Christopher Lee Saunders The topological rigidity phenomenon regarding Hamiltonian diffeomorphisms.

Alessandro Portaluri Bifurcation theory of semi-Riemannian geodesics.

Joa Weber Floer homology and the heat flow.