Abstract

We will deal with the Sobolev regularity of a flow $X : [0,T] \times \mathbb{R}^n \to \mathbb{R}$ associated to a non-smooth vector field $b : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$, i.e. the solution of the Cauchy problem

$$\begin{cases} \partial_t X(t,x) & = b(t, X(t,x)) \\ X(0,x) & = x \end{cases} \quad t \in [0,T], \, x \in \mathbb{R}^n. \tag{P}$$

We are going to discuss assumptions on vector field b in order that (P) admits existence and uniqueness and the associated flow X admits a Sobolev regularity, that is, if, for a given $p \geq 1$, $X(t, \cdot) \in W^{1,p}_{\mathrm{loc}}(\mathbb{R}^n, \mathbb{R}^n)$ for each $t \in [0,T]$. We will review some well-known results in this topic and we will present some new results which are still in progress and are part of a joint work with S. Nicolussi Golo.