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Title: Parallel spinors on Riemannian and Lorentzian manifolds

The talk describes results in joint articles with Klaus Kröncke, Olaf Müller, Hartmut Weiß, and Frederik Witt.

We say that a Riemannian metric on M is *structured* if its pullback to the universal cover admits a parallel spinor. All such metrics are Ricci-flat. The holonomy of these metrics is special as these manifolds carry some additional structure, e.g. a Calabi-Yau structure or a G_2-structure. All known compact Ricci-flat manifolds are structured.

The set of structured Ricci-flat metrics on compact manifolds is now well-understood, and we will explain this in the first part of the talk.

The set of structured Ricci-flat metrics is an open and closed subset in the space of all Ricci-flat metrics. The holonomy group is constant along connected components. The dimension of the space of parallel spinors as well. The structured Ricci-flat metrics form a smooth Banach submanifold in the space of all metrics. Furthermore the associated premoduli space is a finite-dimensional smooth manifold, and the parallel spinors form a natural bundle with metric and connection over this premoduli space.

Lorentzian manifolds with a parallel spinor are not necessarily Ricci-flat, however the rank of the Ricci tensor is at most 1, the image of the Ricci-endomorphism is lightlike. Helga Baum, Thomas Leistner and Andree Lischewski showed the well-posedness for an associated Cauchy problem. Here well-posedness means that a (local) solutions exist if and only if the initial conditions satisfy some constraint equations.

We are now able to prove a conjecture by Leistner and Lischewski which states that solutions of the constraint equations on an n-dimensional Cauchy hypersurface can be obtained from curves in the moduli space of structured Ricci-flat metrics on an $(n-1)$-dimensional closed manifold.