

Fractal dimensions and Diophantine approximations

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We will discuss some problems of diophantine approximations related to fractal geometry. We will discuss in particular some results on geometric properties and the fractal structure of the classical Markov and Lagrange spectra - in particular about the Hausdorff dimensions of intersections of these spectra with half-lines: they always coincide and may assume (depending on the choice of the half-line) any real value in the interval $[0, 1]$.

We will also discuss a recent work in collaboration with Yann Bugeaud on (Lebesgue) typical real numbers from the point of view of diophantine approximations - we will see that, in a certain sense (related to Khintchine's theorem) there are no typical real numbers from the point of view of diophantine approximations, and we will compute fractal dimensions of certain exceptional sets related to problems of this kind.

References

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