Abstract. Filling problems are an important class of problems in quantitative geometry. They arise in geometric measure theory and geometric group theory, but often with different motivations; geometric measure theory partly arose from problems about the existence and regularity of minimal surfaces, while geometric group theory uses filling problems to study the large-scale geometry of a space.

The standard filling problem in geometric group theory is to find the asymptotic growth of the Dehn function. The Dehn function of a space is the minimal function $\delta(\ell)$ such that any closed curve of length ℓ is the boundary of a disc of area $\delta(\ell)$, and its rate of growth can reflect aspects of the geometry of the space, such as negative or nonpositive curvature.

In this course, we will explore some of the connections between a geometric measure theory approach to filling problems and a geometric group theory approach. One of our main tools will be the asymptotic cone of a space, a way of viewing a space "from infinity" which captures the large-scale geometry of a space. Asymptotic cones often have complicated geometry; they include Carnot spaces, $\mathbb{R}$-trees, and subsets of $\mathbb{R}$-buildings, and geometric measure theory gives us powerful tools to study such strange spaces.

Topics covered will include: Asymptotic cones and filling problems in negatively-curved and non-positively curved spaces — connections between the asymptotic rank of a space and its filling problems. Symmetric spaces and buildings — applications to arithmetic groups and the geometry of symmetric spaces. Nilpotent groups and Carnot spaces — filling cycles by using approximations at many scales.