Irreducibility of generalized Schur Polynomials

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Abstract

Let $a \geq 0, a_0, a_1, \cdots, a_n$ be integers. Let

$$f_a(x) = \sum_{j=0}^{n} \frac{a_j x^j}{(j+a)!}.$$

Schur(in 1929) proved that $f_0(x)$ with $|a_0| = |a_n| = 1$ is irreducible $\forall n$. In particular, when $a = 0, a_0 = a_1 = \ldots a_n = 1$, $f_0(x) = 1 + x + \frac{x}{2!} + \cdots + \frac{x^n}{n!}$, the *truncated Maclaurin Series* for e^x, is irreducible. This result does not follow from the well-known *Eisenstein Criterion*.

Schur's result has been generalized by many authors by using $p-$adic methods of Coleman and Filaseta. In this talk, I will give a survey of the some of these results and prove some results on the irreducibility of $f_a(x)$ and family of generalised Hermite-Laguerre polynomials, combining $p-$adic methods with the greatest prime factor of the product of consecutive terms of an arithmetic progression and results from prime number theory.