PRIME NUMBERS IN INTERVALS OF LOGARITHMIC LENGTH

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Let X be large. We will first give a new estimate for the integral moments of primes in short intervals of the type $(p, p+h]$, where $p \leq X$ is a prime number and $h = o(X)$. Then we will use this to prove that for every $\lambda > 1/2$ there exists a positive proportion of primes $p \leq X$ such that the interval $(p, p+\lambda \log X]$ contains at least a prime number, with an explicit bound for the proportion. As a consequence we improve Cheer and Goldston's result (1987) on the size of real numbers $\lambda > 1$ with the property that there is a positive proportion of integers $m \leq X$ such that the interval $(m, m + \lambda \log X]$ contains no primes. We also prove other results concerning the moments of the gaps between consecutive primes and about the proportion of integers $m \leq X$ such that the interval $(m, m + \lambda \log X]$ contains at least a prime number. We discuss similar results, assuming the validity of the Riemann Hypothesis and of a form of the Montgomery pair correlation conjecture.

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